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A THEOREM IN PLANE CUBICS.

By Prof. F. H. Loud, Colorado Springs, Colo.

Let a right line meet a cubic curve in the three points P, Q, R; from each of these points let a line be drawn, viz: From P a line meeting the curve in G and L, from Q one meeting it in H and M, from R one meeting it in K and N. Join these six points by twos as follows: Let the line KM meet the curve in A and meet the line GL in a; let GN meet the curve in B and meet HM in b; let HL meet the curve in C and meet KN in c. Then will (1) the points a, b, c lie on a right line, and (2) the points A, B, C lie on a right line, and (3) the lines abc, ABC, PQR meet in a point.

(1) In the following arrangement:

$$\begin{array}{ccc} P & Q & R \\ L & H & c \\ G & b & N \\ a & M & K, \end{array}$$

the letters placed in any horizontal row represent points which by the foregoing hypotheses lie in one right line; and so also do the letters which are in one vertical column. The columns therefore represent a cubic system and the rows a quartic system intersecting the former in twelve points. The given cubic curve with the line joining b and c forms another quartic system passing through eleven of these points. But a quartic passing through eleven fixed points on a cubic must pass through a twelfth (Salmon, H. P. C., p. 20, § 33); hence the last-named quartic system passes through a. But a cannot be a point of the cubic curve (which has already nine known points in common with the cubic system of lines), hence it lies on the line bc.

(2) Let the line LH meet the curve in C and the line GN meet it in B; also let the line BC meet the line MK in A. If it is proved that A is on the curve, the second part of the above theorem is demonstrated. In the following arrangement:

$$\begin{array}{cccc} L & H & c & C \\ G & b & N & B \\ a & M & K & A, \end{array}$$

the lines of one row or of one column represent collinear points as before, and we again have a cubic system intersected by a quartic; also the line *abc* and the cubic curve which passes through the points *BC*, *GHK*, *LMN* make up a quartic through eleven of these points. Hence this quartic passes through the twelfth point *A*, and as this cannot be a point of the line which intersects the cubic system in the three points *a*, *b*, *c*, it is a point of the cubic curve.

(3) Finally consider this last arrangement,

$$\begin{array}{cccc} P & Q & R & \\ L & H & c & C \\ G & b & N & B \\ a & M & K & A, \end{array}$$

in which as before the letters of one row or of one column represent collinear points. We have here two quartic systems of lines intersecting not only in the fifteen points of the diagram, but also in the point in which the line *PQR* meets *ABC*. The cubic curve and the line *abc* make up another quartic, passing through the fifteen points of the diagram, thirteen of which are independent. This last quartic system therefore passes through the sixteenth point above named, and as this cannot lie on the curve, which has twelve points of intersection with either quartic, it lies on the line. Hence the lines *ABC*, *abc*, *PQR* meet in a point.

It is to be observed that the proof of either the first or second part of the above theorem did not require the whole of the hypothesis as stated. Either of these parts might accordingly be stated as a separate theorem with a simpler hypothesis. The reciprocal theorem, concerning tangents to a curve of third class, and the subordinate theorems which may be obtained by placing at infinity some of the points or lines of the diagram, constitute a group of propositions of which a mere mention is sufficient.

COLORADO COLLEGE, *August* 1, 1890.